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PASSIVE IMPLEMENTATION OF MULTIBODY SIMULATIONS FOR HAPTIC DISPLAY

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ABSTRACT

This paper addresses the issue of providing stability guarantees during the implementation of multibody simulations for haptic display. It introduces the concept of a virtual coupling between the multibody simulation and haptic display hardware. This coupling separates the simulation design process from the sampled-data stability issues that arise during its implementation with a haptic display. Necessary conditions for passivity of a 1-DOF haptic display with a discrete-time passive simulation and an arbitrary virtual coupling are given. The implications of discrete-time passivity on numerical integration and collision response algorithms are also addressed.

1. INTRODUCTION

Many of the proposed applications for haptic displays involve the simulation of a complex dynamic system (e.g. aeronautical or medical training). This simulation must then be interfaced with a haptic display so that information about forces and motions can be exchanged haptically between the human user and the virtual environment. To date, however, virtual environments from published works have consisted primarily of haptic primitives, like virtual walls (Colgate, et al., 1993, Salcudean and Vlaar, 1994), masses, and simple textures (Klatzky, et al., 1989, Minsky, 1995). Other implementations include (Zilles and Salisbury, 1995), which describes an approach to the implementation of complex static environments, but does not address the extension to dynamic environments. Gillespie (1996a) describes the implementation of complex dynamic environments whose constraint configurations are known a priori.

Notably missing from the haptics literature are general approaches to the interface between a haptic display and a complex dynamic simulation. This gap can be at least partially explained by the difficulty of providing *stability guarantees* when working with haptic virtual environments. These guarantees are important both because instabilities are potentially dangerous and because they typically destroy the user's sense of immersion. Most existing virtual environments rely on the careful tuning of environment and control parameters to ensure stability. Whenever changes are made to the environment (such as changing the length of an object), these parameters have to be re-tuned. While merely annoying for relatively simple environments, this process becomes impractical for complex ones.

As an avenue for exploring this topic, our group has begun developing hand tool simulations for use in aeronautical training systems. To help motivate this application, consider the recent use of virtual reality (VR) methods to train Space Shuttle support personnel in procedures involving highly specialized hand tools. While some tools used in micro-gravity are quite ordinary, others have unusual shapes and functions (e.g., various tools for emergency repairs). In the current VR training environment, tools are not represented at all, since this inclusion would require simulation of the interactions between virtual objects. For example, one merely points to a bolt that needs to be loosened, and it loosens itself. Clearly, this type of environment is useful for learning a complicated *procedure*, but not a physical *skill*. To develop a physical skill, haptic interaction is a necessary component of the training.

In light of the hand-tool-based environments we envision building, a general purpose multibody simulation incorporates the required range of physical behaviors. Multibody simulations typically address at least a subset of the following characteristics:

- An environment comprising rigid polyhedra, springs, and dampers
- Bilateral (i.e. permanent) constraints between bodies, such as revolute and prismatic joints
- Unilateral constraints, which typically incorporate collision detection, contact point determination, and collision response (including the addition of constraint forces and/or impulses that act to prevent interpenetration or simulate frictional contact)

In the effort to build complex multibody environments for haptic interface, a natural starting point is the physics-based simulation literature. This literature pulls from several different research communities: computer graphics, robotics, and applied mechanics. In recent years, it has produced a variety of modeling and implementation tools for use with multibody computer simulations. However, since these tools were not designed with haptic interface in mind, it is not clear that they will work properly when used in this context. For a review of multibody simulation formalisms and their applicability to haptic interface, see (Gillespie, 1997).

Our own experience has indicated that algorithms which work well with stand-alone simulations often fail when implemented with a haptic display. As in the general case, the cause of this failure is the difficulty of obtaining stability guarantees for any usefully broad class of environments. To address this difficulty, it would be useful to develop a general-purpose software and hardware architecture for real-time haptic interaction between users and complex dynamic simulations. Ideally, this architecture would decouple the haptic display from the multibody simulation, such that stability of the haptic display is not strongly dependent on simulation parameters (while maintaining a useful connection between them). Our goal is to allow a knowledgeable user (but not an expert in haptic display) to design virtual environments while maintaining confidence that the resultant system will be stable.

In this work, we present an approach to the implementation of multibody simulations for use with a haptic display. In Section 2, we introduce the idea of a *virtual coupling* between the manipulandum and the multibody simulation. We show that if the multibody simulation maintains *discrete-time passivity*, the human-manipulandum-simulation system will have desirable stability properties. Sections 3 and 4 discuss the implications that discrete-time passivity has for numerical integration and collision response algorithms, respectively.

2. THE VIRTUAL COUPLING

The low-level performance limitations of haptic interfaces have been well-documented in recent years. For example, in (Colgate and Brown, 1994), the term *Z-width* was introduced as a performance measure to describe the dynamic range of achievable impedances for a given device. This measure was used to evaluate implementations of virtual walls, resulting in Z-width plots consisting of acceptable ranges of wall stiffness and damping. Various models of haptic display hardware, sample-and-hold, and virtual walls have been developed to identify and improve the Z-width associated with virtual walls (Ellis, et al., 1996, Gillespie, 1996b, Salcudean and Vlaar, 1994).

Extension of the virtual wall model to more complicated environments is often not a trivial task. When a haptic display has both translational and rotational degrees of freedom, even relatively simple virtual environments generate instabilities if implemented unwisely. As an example, consider a peg in hole simulation implemented on a 3-DOF planar haptic display (Figure 1).

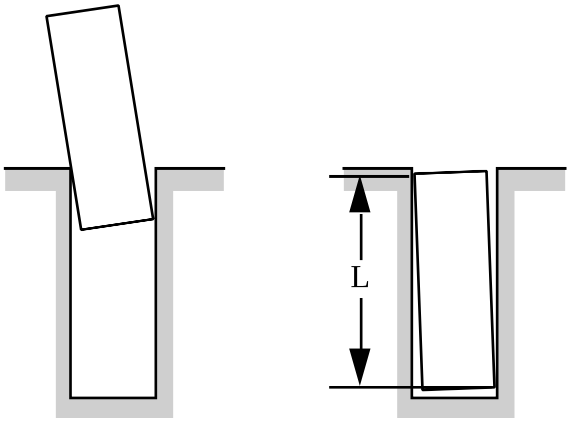


Figure 1. Two configurations of a peg in hole simulation. In this implementation, the massless, perfectly rigid peg tracks the handle of the haptic display. The hole is mechanically grounded and has a contact stiffness of K_C. The vertical distance between the contact points is *L*.

We can obtain the Z-width of the device either through derivation or experimental measurement. This Z-width is expressed as a three-element vector $[K_x\ K_y\ K_\theta]^T$, representing the maximum stiffness implementable across the entire workspace (neglecting for now the role of virtual damping). One approach to keeping the hardware within its Z-width is to model the peg as a perfectly massless rigid body that tracks the handle of the haptic display, while the hole is compliant (and mechanically grounded). If the peg penetrates any surface, a reaction force/torque is computed according to the contact stiffness, K_c, and applied to the handle. In the second configuration of Figure 1, the stiffness matrix of the peg (and thus the handle of the haptic display) is given by:

$$\mathbf{K}_{peg} = \begin{bmatrix} K_c \\ 0 \\ \frac{1}{4} K_c L^2 \end{bmatrix} \qquad (1)$$

There are two problems which make this approach infeasible. First, the rotational and translational components of the stiffness are interdependent (i.e., typically only one component can be set to its maximum stiffness for a given simulation). The other component will either be too compliant or exceed the Z-width of the device. Second, the stability of this simulation is geometry-dependent. For a given contact stiffness, it is always possible to choose a peg length that will result in severe instability. This second problem becomes even worse as simulations become more complicated, due to the increase in the number of parameters which strongly affect stability. A successful implementation needs to utilize the full Z-width of the device, while maintaining stability independent of simulation geometry.

In (Colgate, et al., 1995), a more general approach to the implementation of complex virtual environments was developed. While the analysis was restricted to environments with linear dynamics, an important new concept was incorporated: *the virtual coupling*. Shown in Figure 2 as a multi-dimensional spring and damper, the virtual coupling allows the environment design to be separated from the sampled-data control issues associated with haptic display.

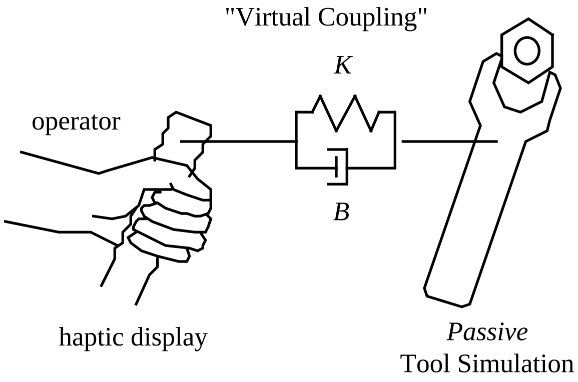


Figure 2. Conceptualization of a virtual coupling in a hand tool simulation. The spring and damper form a connection between the haptic display handle and the multibody simulation, transmitting force and motion between them.

The conceptual separation of the virtual environment from the rest of the haptic display system allows for a rigorous analysis for a large class of simulations. This separation limits the impedance range the haptic display hardware has to render, allowing stability conditions to remain relatively independent of simulation parameters. In the analysis that follows, we extend previous passivity theory to non-linear systems. It will be shown how the virtual coupling allows the full Z-width of the device to be utilized, while removing the effects of geometry on stability. As in previous work, our primary analytical tool is passivity, which provides useful insight about why the haptic display becomes unstable. Figure 3 shows a virtual coupling implementation of a 1 DOF haptic display.

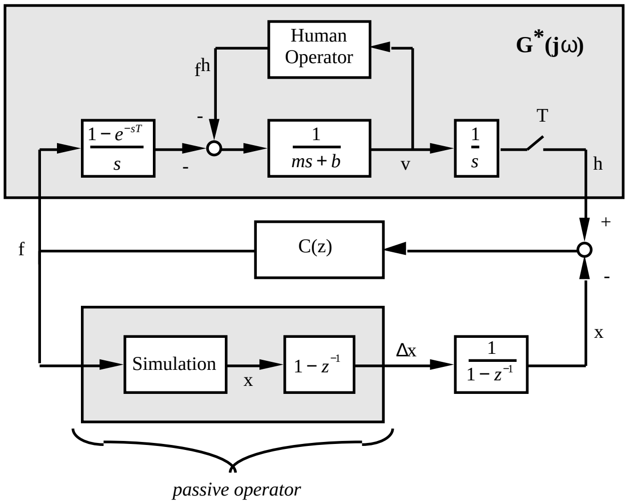


Figure 3. Model of a haptic display with a discrete-time passive virtual environment simulation. $G^*(j\omega)$ represents the dynamics of the sample/hold, mechanism, and human operator, while $C(z)$ is the virtual coupling. Parameters m and b are the mass and damping of the haptic display mechanism, T is the sample time, h is the position of the handle, x is the position of the environment side of the virtual coupling, and f is the virtual coupling force.

The goal of this analysis is to derive the conditions under which the haptic display handle appears passive to the human operator. Under these conditions, the device cannot generate energy continually over time, making actuator-driven instabilities impossible. It is assumed (without explanation for now) that the simulation is *discrete-time passive* (i.e., the operator that maps f to Δx is a passive one). The model of the manipulandum and sample-and-hold is the same as the one used in previous passivity work (Colgate and Schenkel, 1997). It captures the sampled data nature of the haptic display hardware as well as a simple model of the mechanism impedance. The analysis follows a four-step process, resulting in necessary (but not sufficient) conditions for the passivity of the haptic display:

1) Replace the human operator with an arbitrary linear time-invariant passive (ALTIP) impedance. This step does not reflect an assumption about the actual human dynamics, but rather a tool for demonstrating device passivity.
2) Obtain passivity conditions for the mapping from Δx to f (through the virtual coupling and manipulandum).
3) Assume the mapping from f to Δx (through the virtual environment) is a passive one. Thus, the passivity conditions from Step 2 are also sufficient stability conditions for the whole system.
4) The sufficient stability conditions from Step 3 are also necessary passivity conditions for the haptic display, due to the stable connection with an ALTIP impedance.

While not providing a passivity guarantee, this analysis does provide minimum conditions which the haptic display must meet in order to appear passive to the human operator. It should be noted that if the virtual environment dynamics were linear, as in (Colgate, et al., 1995), or if in Step 1 the human operator were replaced with an arbitrary passive operator (and Steps 2-4 were reworked accordingly), then the necessary passivity conditions of Step 4 would also be sufficient passivity conditions.

For Step 2, it can be shown that the condition under which the mapping from Δx to f is passive is given by:

$$b > \frac{T}{2}\frac{1-\cos\omega T}{\mathrm{Re}\left\{\dfrac{1-e^{-j\omega T}}{C\left(e^{j\omega T}\right)}\right\}} \qquad (2)$$

Again, if these conditions are met, then the human/haptic display system will be stable, and the haptic display will meet necessary conditions for passivity. This result is useful because it allows us to evaluate the effectiveness of different virtual couplings. As an example, consider the spring and damper coupling implementation mentioned above, whose continuous-time transfer function is given by:

$$C(s) = K + Bs \qquad (3)$$

where K is the virtual stiffness, B is the virtual damping, and s is the Laplace variable. The discrete version of this transfer function depends on the mapping used between continuous and discrete time. For example, we can calculate the discrete-time transfer functions for a backwards difference and Tustin's mapping, respectively:

$$C_{BD}(z) = K + \frac{B}{T}\left(1-z^{-1}\right) \qquad (4)$$

$$C_{Tustin}(z) = K + \frac{2B}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \qquad (5)$$

where T is the sample time and z is the pulse variable. Substituting (4) and (5) into (2), we obtain two different passivity conditions for the two different virtual couplings. For the spring/damper using the backwards difference, the passivity condition for the haptic display is:

$$b > \frac{KT}{2} + B \qquad (6)$$

This result indicates that for a given mechanism damping level and simulation update rate, the virtual stiffness and damping of the coupling can be chosen such that *any* discrete-time passive simulation will meet necessary conditions for device passivity. The simulation can be based on any physics-based modeling technique, whether linear or non-linear, as long as it obeys discrete-time passivity. By comparison, the passivity condition for the spring/damper using Tustin's mapping is:

$$b > \left(\frac{KT}{2} + B\right)\frac{1+2\lambda\cos\omega T+\lambda^{2}}{2\left(1+\cos\omega T\right)\left(1+2\lambda\cos\omega T\right)} \qquad (7)$$

$$\lambda = \frac{KT-2B}{KT+2B}$$

This expression indicates that infinite physical damping is necessary at the Nyquist frequency, making (5) a poor candidate for a virtual coupling. Future research will try to optimize the virtual coupling through the use of an idealized transmission line.

The use of a virtual coupling like (4) allows us to focus the design process on the multibody simulation, rather than sampled-data stability issues. The only requirement, from a passivity point of view, is that the numerical methods in the simulation be discrete-time passive. This change in focus is desirable because discrete-time passivity is significantly easier to evaluate than sampled-data passivity. Section 3 will address the implications of discrete-time passivity on numerical integration methods, while Section 4 will examine collision response.

3. DISCRETE-TIME PASSIVE NUMERICAL INTEGRATION

When modeling complex multibody systems, the result is typically a set of non-linear differential equations with no known analytic solution. Researchers must resort to using a numerical method to approximate the solutions to this set of equations. Over the years, several different measures of numerical method "goodness" have been developed: accuracy, stability, conservation properties, and computational efficiency.

In this analysis, we will concern ourselves with stability, conservation properties, and computational efficiency. While accuracy is important, and any algorithm that we implement will need to meet some standard of accuracy, our primary concern is to understand how the conservation properties of numerical integrators are related to the discrete-time passivity properties mentioned in Section 2. This connection will allow us to explore some of the implications that discrete-time passivity has on the choice of numerical integration method. Section 3.1 will establish the connection between the energy conservation properties of numerical integrators and their discrete-time passivity, while Section 3.2 will examine the tradeoffs between implicit and explicit numerical integrators when used for a haptic display simulation.

3.1 Energy conservation and discrete-time passivity

A simple example will illustrate the concepts mentioned above. Consider the haptic display of a particle (mass m)

constrained to move along a line, with position *x(t)* and velocity *v(t)*. Assuming the only force acting on the simulated mass is that provided by the human operator, the continuous-time equation of motion is:

$$\dot{v}(t) = \frac{f(t)}{m} \tag{8}$$

where *f(t)* is the virtual coupling force applied to the mass and $\dot{v}(t)$ is the acceleration of the mass. It should be noted that a continuous-time system of this type inherently obeys two conservation laws: conservation of momentum and conservation of energy. From Newton's 2nd law, the change in momentum of the mass exactly equals the total impulse applied by the external force over any time period, and the work done by the external force exactly equals the change in kinetic energy. Even for this simple example, however, numerical integration methods do not necessarily yield solutions that observe these basic conservation laws of mechanical systems.

One common approach to numerical approximation for this type of system begins by breaking the single second-order differential equation into two first order differential equations, and then using a single-step numerical integrator. The general form of this type of integrator is given by:

$$\begin{bmatrix} v_k \\ x_k \end{bmatrix} = f\left(\begin{bmatrix} v_{k-1} \\ x_{k-1} \end{bmatrix}\right) + \begin{bmatrix} g_1(f_k) \\ g_2(f_k) \end{bmatrix} \tag{9}$$

where *f*, g_1, and g_2 encompass the behavior of the numerical method. While we can express the momentum and energy balances for this system, it is important to remember that these conservation laws are arbitrary when discussed in discrete-time. For example, we can compare the total impulse applied by the external force with the change in momentum of the mass. While the change in momentum is well defined from *k-1* to *k*, calculating the total impulse requires an estimation of the force, $\bar{f}$, across the time step:

$$mv_k - mv_{k-1} = T\bar{f} \tag{10}$$

Similarly, the change in kinetic energy of the mass is well defined, while work done requires a force estimate for the time between *k-1* and *k*. If we select f_k as the force estimate, however, we can establish a connection between a numerical integrator's energy conservation properties and discrete-time passivity. The net energy gain as a result of the numerical integrator is given by the difference between kinetic energy gain and work done by the external force:

$$\begin{aligned} E_k &= \Delta K(t_1, t_2) - W(t_1, t_2) \\ &= \frac{1}{2} m(v_k)^2 - \frac{1}{2} m(v_{k-1})^2 - f_k \; (x_k - x_{k-1}) \end{aligned} \tag{11}$$

If E_k is non-positive, then the gain in kinetic energy of the mass is less than or equal to the work done by the external force (i.e., the integrator does not add excess energy to the mass):

$$\frac{1}{2} mv_k^2 - \frac{1}{2} mv_{k-1}^2 - f_k \; (x_k - x_{k-1}) \le 0 \tag{12}$$

If E_k is non-positive at each integration step, then there can be no net energy growth E_g over time:

$$E_g = \sum_{i=1}^{N} \left(\frac{1}{2} mv_i^2 - \frac{1}{2} mv_{i-1}^2 \right) - \sum_{i=1}^{N} f_i \; (x_i - x_{i-1}) \le 0 \tag{13}$$

Expanding the kinetic energy terms, we find that the total work extracted by the virtual coupling force must be less than or equal to the initial kinetic energy of the virtual mass:

$$-\sum_{i=1}^{N} f_i \; (x_i - x_{i-1}) \le \frac{1}{2} mv_0^2 \tag{14}$$

This statement exactly matches the definition for discrete operator passivity (Desoer and Vidyasagar, 1975). Therefore, any numerical integrator which obeys the energy conservation principle established in (12) will be discrete-time passive. When implemented on a haptic display with a virtual coupling, we are then able to develop passivity conditions for that system. As seen in Section 2, these passivity conditions are independent of the parameters of the virtual environment. The importance of this result is that energy conservation principles have already been explored in great depth for numerical integrators. Particularly when dealing with 3-D rotations, creating a numerical integrator that conserves energy is not a trivial task. For example, (Simo and Wong, 1991) demonstrates that the implicit mid-point method is energy conserving in force-free 3-D rotations. It has yet to be demonstrated whether or not this method exhibits the correct energy growth for motions with externally applied forces.

3.2. Implicit vs. explicit numerical integrators

While our theoretical development is not complete, one important characteristic of a numerical integrator we have identified is whether it is implicit or explicit. A numerical operator is implicit if its inputs at the *kth* step depend on the outputs at the *kth* step. Since the virtual environment output immediately affects the input (via the virtual coupling), the integrator will be implicit unless it has at least a one time step delay. This characteristic has important ramifications both for discrete-time passivity and for software implementation. As a means of exploring how implicitness/explicitness affects discrete-time passivity, let us return to the point mass example of the previous section. For the sake of simplicity, we will use Euler's method as the numerical integrator under question. The implicit form is given by:

$$v_k = v_{k-1} + \frac{T}{M} f_k \qquad (15)$$
$$x_k = x_{k-1} + T v_k$$

Straight forward algebra reveals the energy balance:

$$\Delta K_k - f_k \Delta x_k = -\frac{1}{2} m\left(v_k - v_{k-1}\right)^2 \le 0 \qquad f, x, v \qquad (16)$$

From (12), this result implies that the implicit Euler method is discrete-time passive. By comparison, the explicit form of Euler's method is given by:

$$v_k = v_k + \frac{T}{M} f_{k-1} \qquad (17)$$
$$x_k = x_{k-1} + T v_{k-1}$$

and its energy balance is given by:

$$\Delta K_k - f_k \Delta x_k = \frac{1}{2} m\left(v_k + v_{k-1}\right)^2 - m v_{k-1}\left(v_{k-1} + v_{k+1}\right) \qquad (18)$$

Clearly, the change in energy can be positive for this numerical integrator (e.g., if $v_{k-1}>0$ and $v_{k+1}<-v_{k-1}$). Thus, it is not discrete-time passive. Given the developments of Section 2, it would appear that the implicit Euler method is a better choice of numerical integrator for implementation on a haptic display. Our research to date indicates that several implicit integration

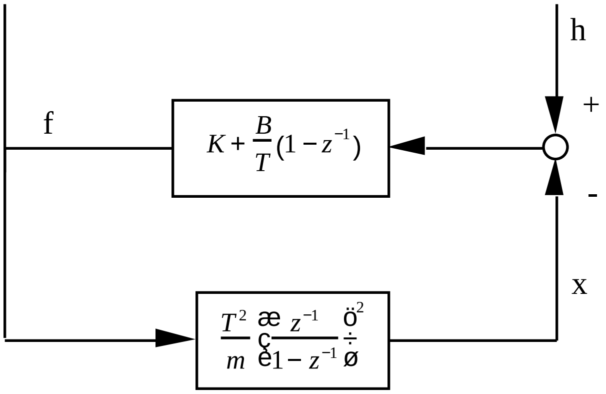
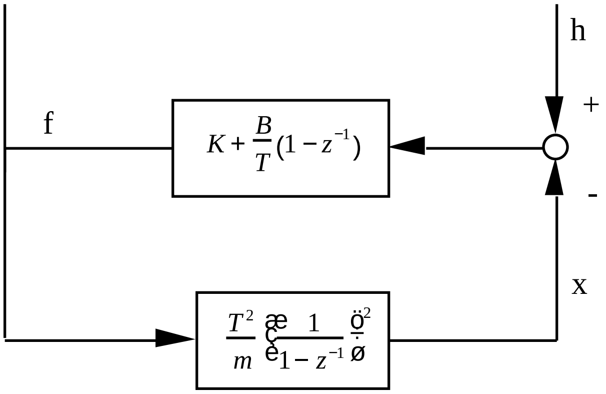


Figure 4. Backwards difference spring-damper virtual coupling and virtual mass integrated with the explicit Euler method.



complete decoupling between passivity and environment parameters, this approach shows promise. Our experience to date is that implementations using the explicit Euler method seem to work reliably for a wide range of simple environments. While not as robust as the implicit version, it does function adequately for a range of environment parameters. It is unclear at this point whether this characteristic will extend to more complicated simulations.

4. DISCRETE-TIME PASSIVE UNILATERAL CONSTRAINT ENFORCEMENT

The other major component of a multibody simulation is unilateral constraint enforcement, which occurs in three steps: collision detection, contact point determination, and collision response. The analysis of this section addresses collision response and its effect on discrete-time passivity. See (Chang and Colgate, 1997) for details on collision detection and contact point determination. Section 4.1 will introduce some basic concepts of unilateral constraints. Section 4.2 shows that energy conservation in collision response, in conjunction with an energy conserving numerical integrator, leads to a discrete-time passive multibody simulation algorithm.

4.1 Basic concepts of unilateral constraints

A collision between two bodies is an extremely complicated event, often featuring fast time constants, deformations of the bodies, shock waves, vibrations, and energy lost to friction and plastic strain. In modeling such a system, the goal is to create the simplest model possible that captures all the "relevant" behavior. In many situations, if the bodies involved do not deform very much, a rigid-body assumption is used. This assumption greatly simplifies the dynamics of the system, and often produces useful results for the analyst.

It is important to remember, however, that *perfectly* rigid bodies do not, in fact, exist. By basing our model on a fictional entity, the analyst is forced into postulating a set of dynamical rules that govern their behavior. For example, it is generally accepted that rigid bodies should be subject to Newton's laws of motion (i.e., they should obey conservation of momentum). These laws alone are sufficient to determine the motion of a rigid body system as long as there are no collisions between bodies. If collisions are allowed to occur, we must make additional assumptions. The following assumptions are typically made in multibody simulations:

- To prevent interpenetration of the bodies involved in the collision, body velocities must change instantaneously. Since finite forces cannot instantaneously change a body's velocity, the collision must generate an *impulse*. An impulse is an infinite force that lasts for an infinitesimal period of time, and whose time integral is non-zero.
- Because a collision takes an infinitesimal period of time to occur, the body configurations will be assumed constant throughout the collision.
- Due to the instantaneous nature of collisions, the effect of non-impulsive external forces on body velocities will be neglected during the course of the collision.

To obtain a full set of motion equations governing collisions, additional assumptions must be made. These additional equations are derived not from Newton's laws or other fundamental principles, but rather from empirical models used to approximate the complex behavior of collisions between bodies. Usually, these models try to predict how much energy is lost due to plastic deformation and friction during the collision. It is through the arbitary nature of the contact model that problems can occur in haptic display.

4.2. Discrete-time passive collision response

Recall from (14) the work-energy expression for a discrete-time passive numerical integrator:

$$\sum_{i=1}^{k} f_i(x_i - x_{i-1}) \geq -\mathrm{E}_0 \qquad (20)$$

where E_0 is the initial kinetic energy stored in the system. When this system undergoes a collision, the stored energy changes by ΔK, depending on the restitution and friction models, and resulting in a final storage of E_c. If we show that a given collision response algorithm cannot create energy, then ΔK will be negative and the following relationship applies:

$$0 \leq E_c \leq E_0 \qquad (21)$$

Assuming they are processed with the same algorithm, additional collisions will further reduce E_c. The work-energy expression for the integrator/collision response algorithm becomes:

$$\sum_{i=1}^{k} f_i(x_i - x_{i-1}) \geq -\mathrm{E}_\mathrm{c} \geq -E_0 \qquad (22)$$

Thus, any collision response algorithm which cannot generate energy will comply with the definition of discrete-time passivity. The importance of this result is that it can serve as a guide in choosing models of restitution and friction.

Energy growth due to poorly formed contact models has been observed by many researchers. The most cited example involves the interaction between Newton's model of restitution and Coulomb friction (Lötstedt, 1984, Smith, 1991, Wang and Mason, 1992). Even in planar configurations, the interaction between the two models results in energy growth. Routh's method (1905), by contrast, has been analytically demonstrated to conserve energy in all planar configurations (Wang and Mason, 1992). Extension of the algorithm, which uses Poisson's hypothesis as a restitution model, to three dimensions is addressed in (Keller, 1986, Mirtich and Canny, 1994). Our research group has implemented Routh's method in a planar impulse-based simulation for haptic display, with good results from the point of view of passivity . However, we have not implemented the Newton/Coulomb contact model, so it is possible the energy growth seen in simulation is insignificant in practice.

5. SUMMARY & FUTURE WORK

This paper has addressed how to provide stability guarantees during the implementation of multibody simulations for haptic display. In particular, we introduced the concept of a virtual coupling between the multibody simulation and haptic display hardware, allowing a rigorous sampled-data passivity analysis. Through the analysis, we found that the virtual coupling separates the simulation design process from the sampled-data stability issues that arise during its implementation with a haptic display. Instead, the simulation design process focuses on discrete-time passivity of the numerical methods used in the multibody simulation. We also discussed the implications of discrete-time passivity on numerical integration algorithms, establishing energy conservation as a property that leads to discrete-time passivity and highlighting the differences between explicit and implicit integrators. We briefly outlined the discrete-time passivity issues associated with collision response algorithms.

Our research group has implemented a multibody simulation on 1 and 2-DOF haptic displays, and is in the process of porting the simulation over to a 4-DOF system. Initial experiments on the 1 and 2-DOF devices look promising, and have identified a number of subtle difficulties associated with real-time multibody simulations (Chang and Colgate, 1997). Future directions for work with the virtual coupling include extension of the theory to multiple degrees of freedom (3-D rotations in particular) and the use of transmission lines as virtual couplings. Refinement of the numerical methods will focus on performance quantification of explicit integrators and reliable models of friction and restitution.

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On page 2, Equation (1) should be:

$$\mathbf{K}_{\mathbf{peg}} = \begin{bmatrix} K_c \\ 0 \\ \frac{1}{2} K_c L^2 \end{bmatrix}$$