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TOWARD GUARANTEED STABILITY IN THE HAPTIC DISPLAY OF VIRTUAL ENVIRONMENTS

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ABSTRACT

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A haptic display is a device that allows human operators to manipulate and feel virtual environments. An important issue in the haptic display of virtual environments is stability. Often, unintended oscillations of the human/display system are found to occur. This dissertation addresses the problem of *guaranteed stability* in the haptic display of virtual environments. The approach taken to this problem is based upon ensuring not only discrete time stability, but also discrete time passivity. The introduction and background of this work are included in Part I.

In Part II, we consider the effect of unilateral nonlinearities on the stability of discrete-time control systems, a problem of some importance in haptic display. For example, a virtual wall is a simple form of *unilateral constraint.* Sufficient conditions for the stability of linear, shift-invariant systems in feedback with unilateral nonlinearities are derived. The derivation follows a method originally presented by Mitra in which the existence of periodic oscillations is first assumed, then conditions leading to a contradiction are found. The results developed here are presented graphically in the Nyquist plane, allowing direct comparison to other well-known criteria, such as Tsypkin's Condition. It is shown that the new criterion is much less conservative.

In Part III, we show that A-stability relates to the passivity of the discrete time

simulation. If A-stable methods are used to map linear time-invariant systems to discrete time, the results will be discrete time passive. Furthermore, the need for real-time simulations of complex dynamic systems in haptic display suggests that simulations may need to be distributed across a network of processors. A parallel simulation structured in a physically meaningful way allows the user to apply the powerful insights and methods of physical network theory. We show that, by modeling junctions in the simulation network using transmission lines, the inherent delays are accommodated, and discrete time passivity is guaranteed.

In conclusion, we are able to construct A-stable simulation networks through the transmission line model and A-stable integration technique. We believe that this is a promising approach to the real-time, A-stable simulation of multibody systems.

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