Lead Compensator Design to Meet Steady-State (or Response Speed) and Phase Margin Specs

Plant: $H(s)$
Lead compensator: $D(s) = K (Ts + 1)/(\alpha Ts + 1), \alpha < 1$
OLTF of the UFS: $D(s)H(s)$

1. Choose K to satisfy the steady-state error requirement. (Compensator pole and zero have no effect on the steady-state response.)

   Alternatively, you could choose K to satisfy a speed-of-response requirement. Such a requirement is that the closed-loop system have a bandwidth (a -3dB magnitude response of the closed-loop system) of a chosen ω_{BW}. To get this, choose K to give a crossover frequency for the OLTF of about $(1/2)\ \omega_{BW}$.

2. Evaluate the phase margin (PM) of the system $KH(s)$.
3. Calculate the amount of phase to add, $\phi_{max} = (PM_{desired} - PM)$ + fudge factor (5-20 deg).
4. Calculate $\alpha = (1 - \sin \phi_{max})/(1 + \sin \phi_{max})$. The *lead ratio* is $1/\alpha$.
5. The maximum phase contribution occurs at $\omega_{max} = 1/(\alpha^{1/2}\ T)$. Choose T so the maximum phase contribution occurs near the crossover frequency ω_c of $KH(s)$, or at a somewhat larger value (since the crossover frequency will increase).
6. Draw the new frequency response of $D(s)H(s)$, check the new phase margin at the new crossover frequency, and iterate on the design until the specs are met.