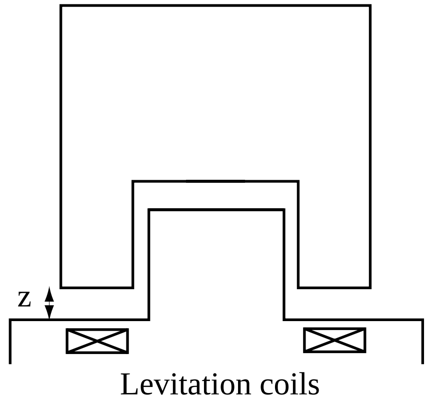
ME 391 Fundamentals of Control Systems
Assignment 1
Due at the beginning of class Friday Jan 14

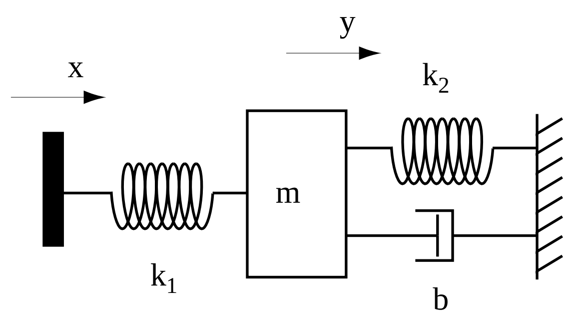
1. At right is a drawing of a maglev train. The levitation force F_L is caused by current i through the levitation coils, and is governed by $F_L = k\,i / (z + 0.05\text{m})^2$, where z is the height of the train above the track in meters. The mass of the train is M, and gravity is g (acting downward).



a. Give the equation for the vertical acceleration z".
b. Choose the state variables $x_1 = z$, $x_2 = z'$, and write the state equations in the form $\mathbf{x}' = \mathbf{f}(\mathbf{x}, i)$.
c. Give the linearized equations of motion $\delta\mathbf{x}' = A\,\delta\mathbf{x} + B\,\delta i$ at a general equilibrium operating point (x_{10}, x_{20}, i_0).
d. Given a desired height of the train above the track x_{10} = 5 cm, find the equilibrium operating point (x_{10}, x_{20}, i_0) if the train's mass is M = 100,000 kg, gravity is g = 9.8 m/s^2, and k = 10 N m^2/A. Give the units.
e. If the output of interest is the height of the train above the track, find the transfer function Z(s)/I(s) at the operating point.

2. The equations of motion for a certain system can be written $d^3y/dt^3 + 3\,y \cos y + 4\,u^3 + (dy/dt)^2 = 50$.
a. Give the state variables $\mathbf{x} = (x_1, x_2, ..., x_n)$, and write the equations of motion in the form $d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, u)$.
b. Find the equilibrium operating point $(\mathbf{x}_0, u_0)$ if we would like to stabilize the system at $y = \pi$.
c. Write the linearized equations of motion $d(\delta\mathbf{x})/dt = A\,\delta\mathbf{x} + B\,\delta u$ for small variations about this operating point.

3. For the mass-spring-damper system at right, find the equation of motion of the mass m, where y is its position, in terms of the input position x.



4. For the circuit shown, find the transfer function $I_3(s)/V_{in}(s)$. Recall Kirchoff's current and voltage laws and the constitutive laws $v = iR$, $v = L\,di/dt$, and $dv/dt = (1/C)\,i$.