Inexpensive Conveyor-Based Parts Feeding

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Abstract

This paper describes a manipulation primitive called toppling—knocking a part over. We derive the mechanical conditions for toppling and describe two applications to conveyor-based parts feeding. Toppling complements previous approaches to conveyor-plane feeding, allowing full three-dimensional parts feeding on a constant-speed conveyor.

Keywords

Parts feeding, toppling, pushing, minimalist robotics

1 Introduction

A recent trend in parts feeding research has been toward the development of inexpensive parts feeders using simple hardware. This approach has been called *minimalist* robotics (Bicchi and Goldberg, 1996). The usefulness of simple hardware for parts feeding comes from taking advantage of the mechanical properties of the parts to assist in feeding and sorting. For example, a centrifuge uses simple rotational motion to sort particles based on density. Unfortunately, feeding complex three-dimensional parts is rarely so simple. The challenge is to develop algorithms for automatic design and control of mechanically simple feeders based on part CAD models, inertial parameters, friction, and restitution.

In this paper we focus on inexpensive conveyor-based methods for parts feeding. Peshkin and Sanderson (1988) described a method for designing a sequence of fixed fences above a conveyor so that a part traveling on the conveyor hits the first fence, aligns to it, drifts off and hits the second fence, aligns, and so on, until at the end of the sequence the part is in a unique orientation. Brokowski *et al.* (1993) modified the shape of the fences to reduce alignment error, and Wiegley *et al.* (1996) gave a complete algorithm for finding fence sequences given the part geometry and center of mass.

Instead of using a sequence of fences, Akella *et al.* (1997) showed that a single rotating fence (a one-joint robot, or "1JOC" for 1 Joint Over Conveyor) was capable of positioning and orienting any singulated polygonal part on a constant-speed conveyor. The 1JOC alternates between pushing the part upstream and letting the part drift with the conveyor. By executing an open-loop series of motions designed for the specific part, the 1JOC brings the part to a unique configuration.

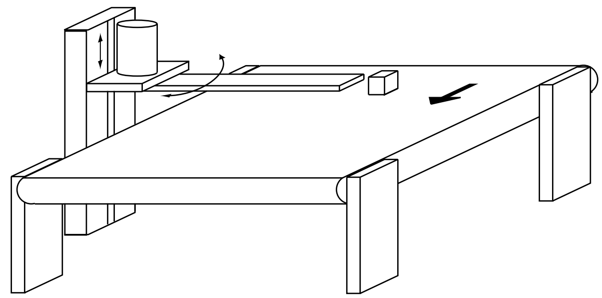

Figure 1: The 2JOC feeds parts on a conveyor by pushing and toppling.

These conveyor-based systems only position and orient the part in the plane of the conveyor. In this paper we investigate a method for *toppling* the parts out of the conveyor plane. Toppling occurs when the conveyor's motion and a pushing contact causes the polyhedral part to topple over an edge to a new support face. Toppling, coupled with the methods above for conveyor-plane feeding, allows full three-dimensional parts feeding on a constant-speed conveyor.

This paper derives the mechanical conditions for toppling and describes two applications of toppling. The first uses fixed overhangs above the conveyor to reduce uncertainty in the part orientation. The second adds a vertical prismatic joint to the 1JOC, creating a 2JOC (Figure 1) which can topple parts in addition to controlling the conveyor-plane position and orientation. The first application is fixed automation; the second uses a very simple, inexpensive two-joint robot.

Related parts feeding research includes work toward the automatic design of bowl feeders (Boothroyd, 1992; Caine, 1994; Berkowitz and Canny, 1996; Christiansen *et al.*, 1996; Berretty *et al.*, 1999) and analysis of the Sony APOS parts feeder (Hitakawa, 1998; Krishnasamy *et al.*, 1996).

2 Toppling

Toppling consists of two phases: rolling (Section 2.1) and settling (Section 2.2). During rolling the robot pushes the part up onto a *toppling edge*, which is perpendicular to the motion of the conveyor, until the center of mass of the part is directly above the edge. During settling the part falls under gravity, lands on a new face, and perhaps continues to roll onto another face before coming to rest.

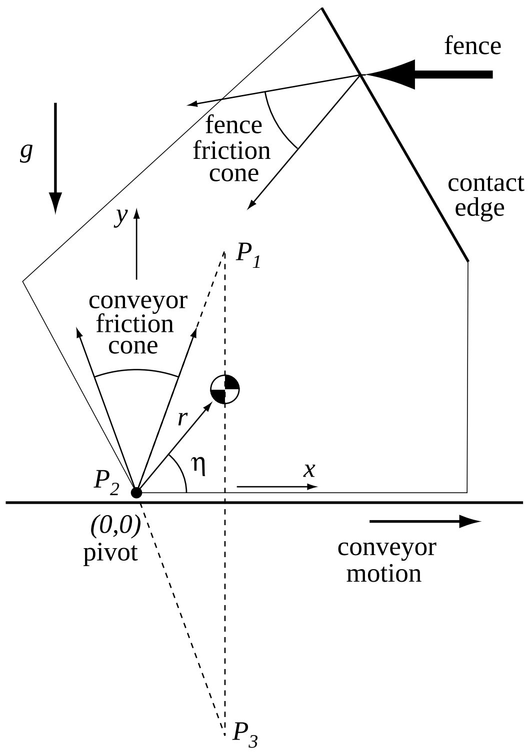

Figure 2: Notation for rolling.

We define two planes: the *toppling plane* and the *conveyor plane*. The toppling plane is a plane orthogonal to the toppling edge. In our analysis of toppling we project the part onto this plane, and the toppling edge projects to a toppling, or pivot, vertex. We assume the projection of the part to the toppling plane is polygonal. The conveyor plane is the plane of the conveyor, and it is orthogonal to the toppling plane. All toppling analysis occurs in the toppling plane; the conveyor plane is relevant when we include pushing motions in this plane with the 2JOC.

2.1 Rolling Conditions

The part rests on a horizontal conveyor moving to the right on the page (Figure 2). We define a frame fixed to the conveyor with origin at the pivot vertex of the part (which moves with the conveyor) with the x-axis aligned with the direction of motion of the conveyor and the y-axis vertical. The center of mass of the part in this frame is at a distance r from the origin at an angle η. The friction coefficients μ_c and μ_f correspond to friction between the part and the conveyor and between the part and the fence, respectively. The corresponding friction cone half-angles are $\alpha_c = \tan^{-1}\mu_c$ and $\alpha_f = \tan^{-1}\mu_f$.

The question is, what fence contact points along the contact edge will result in the part initially rolling over the pivot vertex?

The key construction is shown in Figure 2. Draw a vertical line through the center of mass, extend the right edge of the friction cone at the pivot until it intersects this line, and extend the left edge of the friction cone backward until it intersects this line. This defines a triangle with vertices P_1 at $(r\cos\eta, (r\cos\eta)/\mu_c)$, P_2 at $(0,0)$ (the pivot), and P_3 at $(r\cos\eta, (-r\cos\eta)/\mu_c)$ in the conveyor frame. If the fence is rigid, and the contact force the fence applies to the part makes positive moment about every point in this triangle (i.e., the contact force passes around the triangle in a counterclockwise fashion), then the only quasistatic solution is that the part rolls about the pivot vertex. To guarantee rolling, every force in the fence friction cone must make positive moment about every point in the $P_1P_2P_3$ triangle. In Figure 2, the fence friction cone shown barely satisfies this condition.

The contact friction cone in Figure 2 marginally satisfies the rolling condition, and it is apparent that any higher contact point will also satisfy the condition. Thus the construction of Figure 2 confirms these intuitive properties of rolling:

- "higher" fence contacts tend to produce rolling, while lower contacts result in slipping on the conveyor;
- a larger conveyor friction coefficient μ_c results in a smaller $P_1P_2P_3$ triangle, increasing the set of contact points that produce rolling;
- a center of mass further to the left results in a smaller $P_1P_2P_3$ triangle, increasing the set of contact points that produce rolling.

The construction also shows that the height of the center of mass plays no role in the quasistatic, dry friction rolling conditions.

The analysis of Figure 2 only addresses the instantaneous initial condition for rolling. As the part rolls, it may become wedged or begin slipping on the conveyor. To analyze the gross motion of the part after it begins rolling, we consider two models of the motion of the fence. In the first model, the fence "complies" to the shape of the part in a position-controlled manner, so that the contact point on the part remains constant during rolling. In the second model, the fence remains motionless. The first model is easier to analyze, and it increases the set of parts that can be toppled. The second model requires no motion by the fence.

2.1.1 Position-Controlled Fence

To simplify analysis and to be able to topple parts that might otherwise slip or become wedged, we raise or lower the fence so *the contact point on the part remains constant* as the part rolls. The velocity of the conveyor is known, and given the geometry and initial position of the part, the fence can simply "comply" to the shape of the part in a position-controlled manner. The fence maintains contact until the angle η to the center of mass becomes greater than $\pi/2$. The part then falls to a new stable edge. See Figure 3.[1]

The goal is to find contact points that maintain the rolling condition as the center of mass rolls from its initial angle η_i

[1] If the next clockwise edge contacts the conveyor before η reaches $\pi/2$, that edge is unstable. In this case the next vertex clockwise on the part's convex hull becomes the pivot vertex. For simplicity, we will ignore this case.

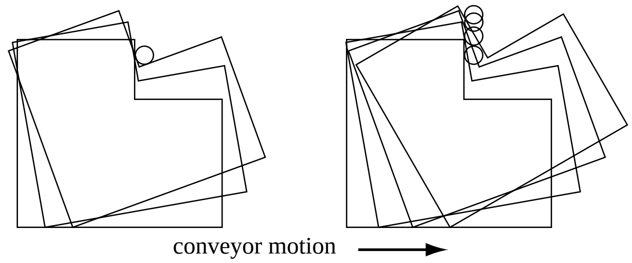

Figure 3: The part on the left becomes wedged or begins slipping with a fence of fixed height. The fence on the right lifts up to maintain a constant contact point on the part, allowing it to topple over.

to its final angle $\eta_f = \pi/2$, at which point the part topples over. As the part rolls, the fence friction cone moves in the conveyor frame. (The $P_1P_2P_3$ triangle also shrinks.) As a result, a contact that causes rolling at η_i may not cause rolling as η increases—the part may begin to slide or wedge.

We have implemented an algorithm which, for each stable resting configuration of the polygon, finds the toppling contacts on each edge. Details of the algorithm can be found in (Lynch, 1999). A result is shown in Figure 4. As expected, increasing conveyor friction increases the range of toppling contacts.

This analysis ensures that any force inside the fence friction cone causes rolling, and is therefore conservative. Instead, we could simply verify that, at all times, there exists a contact force inside the fence friction cone that produces rolling.

2.1.2 Fixed Fence

If the fence remains a fixed height as the part moves on the conveyor, we must find a range of fence heights y that guarantees toppling. The following conditions must also be satisfied: the fence cannot lose contact with the part as it rolls, and rolling must continue if the fence switches contact to a new edge during rolling (the part cannot wedge in a concavity or begin to slip).

2.2 Settling

When the center of mass passes the vertical with respect to the pivot point, the part begins to free fall. The part may simply come to rest on the next edge, or it may continue past this edge and come to rest on a subsequent edge.

The part falls like a pendulum, as shown in Figure 5, until impacting at the next vertex clockwise of the pivot vertex on the part's convex hull. The pre-impact linear velocity of the part's center of mass is $(\dot{x}^-, \dot{y}^-)$, and the angular velocity is $\dot{\eta}^-$, given by

$$\dot{\eta}^- = \sqrt{\frac{2mg(h_0 - h_1)}{I_p}},$$

where m is the mass of the part, g is the gravitational constant, and h_0 and h_1 are the height of the center of mass above the conveyor at the beginning and end of the free fall, respectively. I_p is the inertia of the part about the pivot, where $I_p = m(\rho^2 + h_0^2)$, and ρ is the radius of gyration of

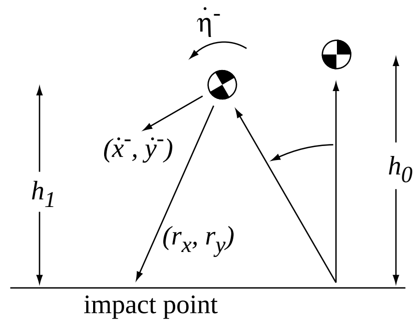

Figure 5: Notation for settling analysis.

inertia of the part measured about its center of mass. The vector from the center of mass to the impact vertex is (r_x, r_y).

To determine the settling edge, we assume a perfectly plastic impact between the impact vertex and the conveyor. This places three constraints on the post-impact velocity. The first two are kinematic, and the third indicates that the impulse passes through the impact point:

$$\begin{aligned} \dot{x}^+ &= r_y \dot{\eta}^+ \\ \dot{y}^+ &= -r_x \dot{\eta}^+ \\ r_x(\dot{y}^+ - \dot{y}^-) - r_y(\dot{x}^+ - \dot{x}^-) &= \rho^2(\dot{\eta}^+ - \dot{\eta}^-). \end{aligned}$$

Solving yields

$$\dot{\eta}^+ = \frac{\rho^2 \dot{\eta}^- + r_y \dot{x}^- - r_x \dot{y}^-}{\rho^2 + r_x^2 + r_y^2}.$$

The pendulum now begins a new free fall stage about the new impact vertex with this initial angular velocity. The part has settled when the post-impact velocity causes immediate re-impact with the previous vertex on successive impacts.

2.3 Toppling Transition Directed Graph

With the rolling conditions and the settling analysis, we can construct the *toppling transition directed graph* for a planar part. Each node of the graph corresponds to a stable resting edge for the part. From each node there is a single arc that leads to the node the part reaches after toppling. This arc is tagged with the contact points on the part (in the case of a position-controlled fence) or the fence heights (in the case of fixed fences) that result in toppling. If no fence contacts can result in toppling from this node, the arc is eliminated. Increasing conveyor friction μ_c can result in the addition of arcs to the graph. Figure 4 shows an example for a position-controlled fence.

Finding a fence plan amounts to searching this graph for a sequence of actions leading from the start node to the goal node.

3 Applications

3.1 Fixed-Height Fences

A sequence of stationary fences over a conveyor can be used to reduce uncertainty in the orientation of a part in

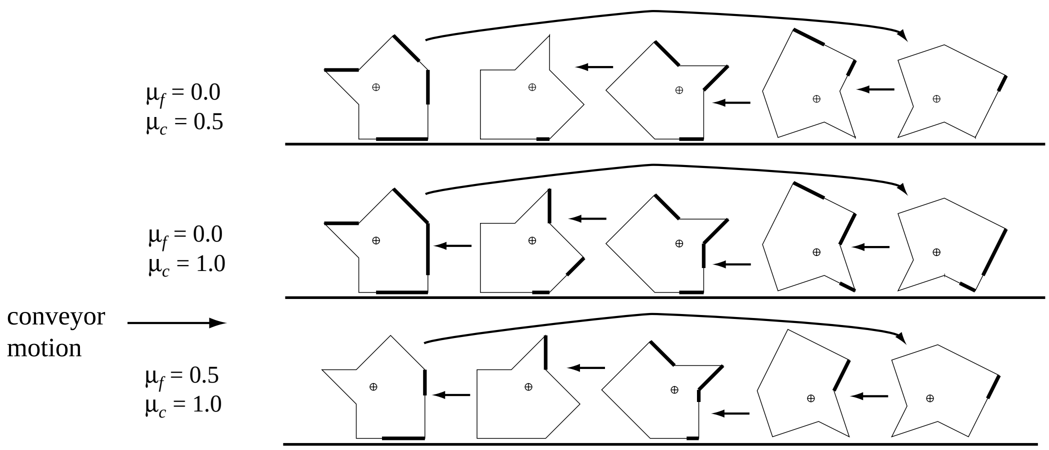

Figure 4: Toppling contacts for a position-controlled fence and different fence and conveyor friction coefficients. The toppling contacts are indicated by heavy lines. As we increase μ_c, the ranges of toppling contacts increase. The arrows indicate a toppling transition directed graph. In the case $\mu_c = 0.5$, there is a single resting configuration from which the part cannot be toppled. The fence would have to lift up on the bottom edge of the part to cause toppling.

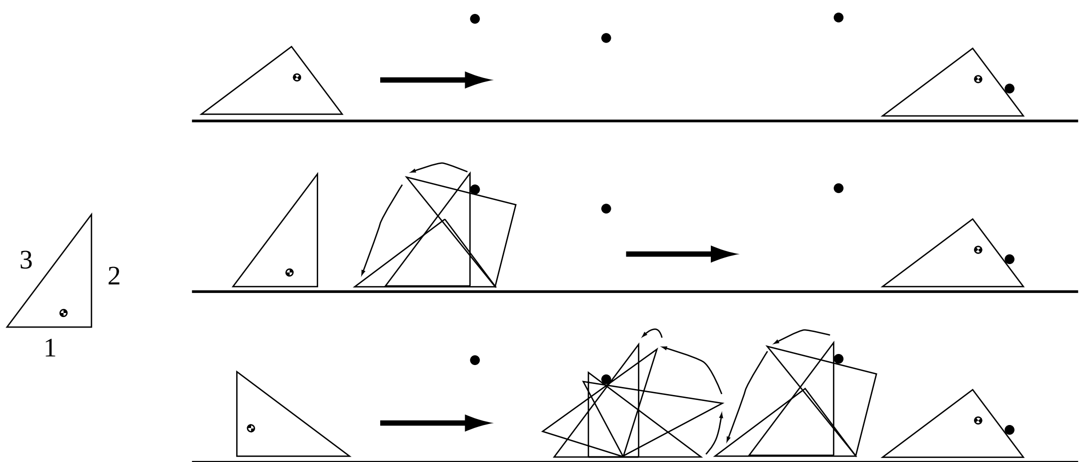

Figure 6: A sequence of fixed-height pegs that eliminates uncertainty in the part's toppling-plane orientation. At each peg, the part either passes underneath or topples to a new edge.

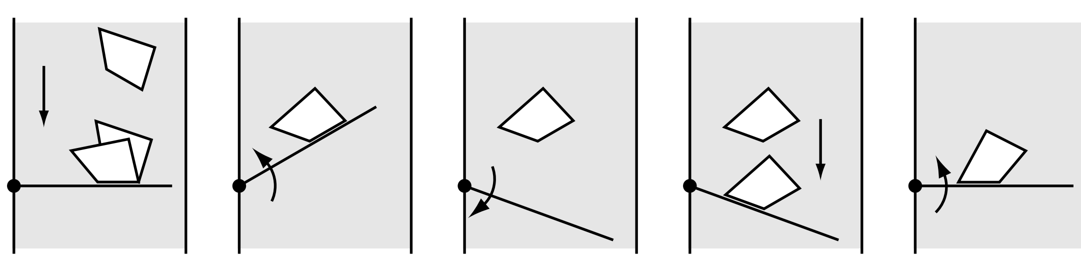

Figure 7: The 1JOC positions and orients parts in the conveyor plane by pushing.

Figure 9: The experimental 2JOC.

the toppling plane. By combining toppling with sensorless approaches to conveyor-plane parts feeding (Akella *et al.*, 1997; Peshkin and Sanderson, 1988; Brokowski *et al.*, 1993; Wiegley *et al.*, 1996), it is possible to construct sensorless 3D parts feeding devices on conveyors.

An example sequence of fixed fences is shown in Figure 6. The part is a 3-4-5 triangle, with edges labeled as shown in Figure 6 and vertices at $(-2,-0.5)$, $(1,-0.5)$, and $(1,3.5)$ with respect to an origin at the center of mass. The friction coefficients are $\mu_f = 0$ and $\mu_c = \tan 30^\circ = 0.577$. A fixed-height peg at a height y above the conveyor, $3.467 < y < 3.881$, causes a triangle at rest on edge 1 to topple to rest on edge 3, while a triangle at rest on edges 2 or 3 passes under the peg. (If $y > 4$, the triangle passes under the peg; if $y > 3.881$, the peg loses contact with the triangle before it has finished rolling.) A peg at $2.361 < y < 2.698$ causes the triangle to topple from edge 2 to edge 1. No peg location will cause toppling from edge 3 to edge 2. Combining these constraints, we find that three fixed-height pegs are necessary and sufficient to bring this triangle to a unique resting edge at the end of the sequence.

These fences are analogous to the overhangs used to topple parts in bowl feeders. In general, fixed-height fences cannot remove all toppling-plane uncertainty.

3.2 2JOC

The 1JOC (Akella *et al.*, 1997) feeds parts in the conveyor plane by using a single revolute robot joint to push parts on the conveyor (Figure 7). By combining part motion on the conveyor with a series of pushing motions, it is possible for the 1JOC to take any polygonal part from any configuration on the conveyor (upstream of the robot) to a single desired configuration in the conveyor plane.

We have augmented the 1JOC with a prismatic joint that allows the fence to move vertically (Figures 1 and 9). We call this system the 2JOC. The goal is to combine toppling with the ability of the 1JOC to perform conveyor-plane feeding, resulting in full 3D parts feeding on a conveyor.

For example, consider the 3x2x1 uniform mass rectangular block of Figure 8. Set $\mu_f = 0$ and $\mu_c = 0.5$. With these values, the toppling transition directed graph is shown. The part can be toppled from face A to face B when the edge AB (adjacent to faces A and B) is perpendicular to the motion of the conveyor and furthest upstream on the conveyor. Then the spatial toppling problem reduces to the planar problem above, with edge AB acting as the pivot vertex. During toppling, the fence uses its vertical prismatic motion to maintain a constant contact point on the part, but does not rotate in the conveyor plane. By sequencing 1JOC pushing and toppling, we can control the 3D configuration of the part.

If $\mu_f = 0, \mu_c > 1.5$ for the block of Figure 8, then any topple is possible; any face is reachable from any other face by toppling. In fact, for any rectangular prism with $\mu_f = 0$, there exists a μ_c^{crit} such that this property holds for all $\mu_c > \mu_c^{crit}$. This property, coupled with the 1JOC feeding property, indicates that any rectangular prism can be taken from any initial 3D configuration on the conveyor to a desired goal configuration, provided μ_c is sufficiently high.

Although the analysis in this paper is sufficient to analyze the 2JOC manipulating rectangular prisms, a number of research issues remain for feeding more general 3D parts. A video of the 2JOC manipulating a rectangular prism can be found at
`http://lims.mech.nwu.edu/~lynch/research/2JOC/`.

4 Conclusions

Toppling is a mechanically simple manipulation primitive that can be used in conjunction with pushing to construct inexpensive conveyor-based parts feeders. This paper derives the mechanical conditions for toppling and describes two applications. Future work should address toppling conditions and motion planning for general three-dimensional parts.

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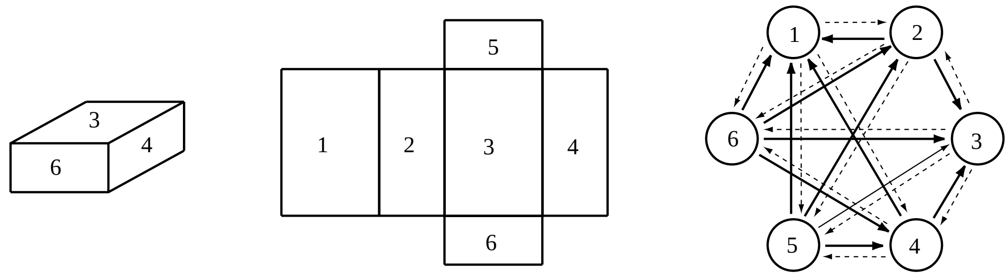

Figure 8: The toppling transition directed graph for a 3x2x1 uniform mass rectangular prism where $\mu_f = 0$ and $0.333 < \mu_c \leq 1.0$. Each node corresponds to a support face. The solid arrows correspond to transitions with feasible toppling contacts; the dotted transitions are unachievable. For $\mu_c > 1.5$, all transitions are possible.

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